Physics Notes

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Class:10+1
Unit: IX
Topic: Behavior of Perfect Gas and Kinetic Theory of Gases

SYLLABUS: UNIT-IX

Equation of state of a perfect gas, work done on compressing a gas.
Kinetic theory of gases-assumptions, concept of pressure. Kinetic energy and temperature; rms speed of gas molecules; degrees of freedom, law of equipartition of energy (statement only) and application to specific heats of gases, concept of mean free path, Avogadro's number.

Q.1. What are assumptions of Kinetic Theory of Gases?

Ans. Assumption of Kinetic Theory of Gases:

The entire structure of the kinetic theory of gases is based on the following assumptions which were first stated by *Classius*.

(1) A gas consists of a very large number of molecules which are perfect **elastic spheres** and are identical in all respects for a given gas and are **different for different gases**.
(2) The molecules of a gas are in a state of continuous, rapid and random motion. They move in all directions with **different speeds**, ranging from **zero to infinity** and obey Newton's laws of motion.
(3) *The size of the gas molecules is very small as compared to the distance between them. Hence* ***volume occupied by the molecules is negligible*** *in comparison to the volume of the gas.*
(4) *The molecules do* ***not*** *exert any force of* ***attraction or repulsion*** *on each other, except during collision.*
(5) *The* ***collisions*** *of the molecules with themselves and with the walls of the vessel are* ***perfectly elastic****.* As such the momentum and the Kinetic energy of the molecules are conserved during collisions.
(6) *Molecular* ***density is uniform*** *throughout the gas*.
(7) *A molecule moves along a straight line between two successive collisions* and the average straight distance covered between two successive collisions is called the **mean free path** of the molecules.
(8) The collisions *are almost instantaneous*, i.e. the time of collision of two molecules is negligible as compared to time interval between two successive collisions.

Concept Example: (for Q no.2)

5 cars have speed 40 km/hr, 60, 60, 60, 80 km/hr.
Find Average, RMS and most probable speed.

Ans. Averageg Speed $= \frac{20 + 60 + 60 + 60 + 80}{5}$ = 56 km/hr

RMS Speed $= \sqrt{\frac{20^2+60^2+60^2+60^2+80^2}{5}}$ = 59.33 km/hr

Most Probable Speed = 60 km/hr as three cars have speed 60 km/hr
speed possessed by maximum number of cars.

Q2. **Explain the following for gas atoms:**

a) Most probable speed

b) Average speed

c) R.M.S. speed

Ans.a) Most Probable speed:

We may define *most probable speed of the molecules of a gas as that speed which is possessed by maximum fraction of total number of molecules of the gas.*

It can be shown that

$$C_{mp} = \sqrt{\frac{2\,k_B T}{m}} \quad (1)$$

b) Mean speed or Average speed:

Mean speed or Average speed is the average speed with which a molecule of the gas moves.

$$C_{av} = \frac{C_1 + C_2 + \ldots .C_n}{n}$$

From Maxwellian speed distribution law, we can show that

$$C_{av} = \sqrt{\frac{2.56\,k_B T}{m}} \quad (2)$$

Where m is mass of each molecule, k_B is Boltzmann constant and *T* is temperature of the gas.

c) Root Mean Square Speed:

Root mean square speed of gas molecules is defined as the square root of the mean of the squares of the random velocity of the individual molecules of a gas.

$$C_{rms} = \sqrt{\frac{C_1^2 + C_2^2 + \ldots .C_n^2}{n}}$$

From Maxwellian speed distribution law, we can show that

$$C_{rms} = \sqrt{\frac{3\,k_B T}{m}} \quad (3)$$

Where the symbols have their usual meaning.

From equations (1), (2) and (3), we find that $C_{mp} : C_{av} : C_{rms} = \sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3}$

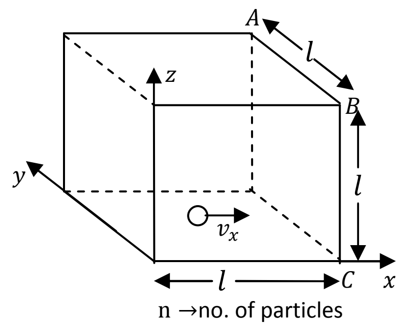
$\therefore C_{rms}$ is maximum and C_{mp} is minimum, out of the three

Q3. a) Derive an expression for pressure of a gas

OR

P.V = $\frac{2}{3}$.E where E is internal energy of a gas.

b) Prove Kinetic Energy per atom = $\frac{3}{2}$.k.T.



Ans.a) $P_x = \frac{Force\ on\ face\ ABCD \times n}{Area\ of\ face\ ABCD}$

$= \frac{\Delta p}{\Delta t} \times \frac{n}{l^2}$

$= \frac{2.m.v_x}{\frac{2l}{v_x}} \times \frac{n}{l^2}$

$$P_x = \frac{m.v_x^2}{l^3} \times n \quad \text{.......... (1)}$$

Similarly, $P_y = \frac{m.v_y^2}{l^3}.n$ and

$P_z = \frac{m.v_z^2}{l^3}.n$

P $= \frac{1}{3}\left(P_x + P_y + P_z\right)$

$= \frac{1}{3} \cdot \frac{m.n.v^2}{l^3}$ $\left[where\ v = \sqrt{v_x^2 + v_y^2 + v_z^2}\right]$

$= \frac{1}{3} \cdot \frac{mass}{volume} \cdot v^2$

$$P = \frac{1}{3} \cdot \rho\, v^2 \quad \text{.......... (2)}$$

OR

P $= \frac{1}{3} \cdot \frac{m.n}{V} \cdot v^2$

P.V $= \frac{1}{3} \cdot n \cdot mv^2$

$= \frac{2}{3} \cdot \left[n\left(\frac{1}{2}.mv^2\right)\right]$

$$P.V = \frac{2}{3} \cdot E \quad \text{.......... (3)}$$

Where E $= n\left(\frac{1}{2}.mv^2\right)$ is total internal energy of gas b)

K.E/atom, $\frac{1}{2}.mv^2$	$= \frac{3}{2}$. k.T

Proof:

As proved $P.V = \frac{2}{3}.E$ .... (3)

Also $P.V = \mu.R.T$ .... (4)

Where μ is number of moles

From (3) and (4)

$$\frac{2}{3}.E = \mu.R.T$$

$$\Rightarrow \frac{2}{3}.E = 1.R.T$$ (Assume one mole of gas)

$$\Rightarrow \frac{2}{3}.\left(N_A.\frac{1}{2}.mv^2\right) = (N_A.k)T$$

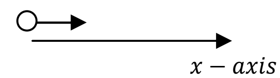
$$\boxed{\frac{1}{2}.mv^2 = \frac{3}{2}.k.T}$$

i.e. K.E/atom, $\frac{1}{2}.mv^2 = \frac{3}{2}$. k.T

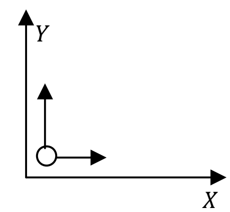
Q4. a) Degrees of freedom for monoatomic, diatomic gas.
b) Law of equipartition of energy.

Ans. a) **<u>Degrees of freedom</u>**:-

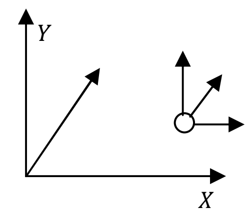
1. A particle moves only along one axis, we say it has one degree of freedom (1-Dimensional motion).



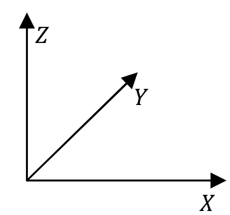
2. A particle can move along a plane, it has 2 degree of freedom (2- Dimensional motion).



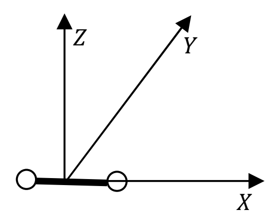
3. Particle can have motion along 3- Dimensional. It has 3 degree of freedom



4. Monoatomic gas:
   (ex- Argon) Atom can move along 3-Dimensional, it has three degrees of freedom.



5. Diatomic gas:
   (ex- O_2 or N_2) Molecular has 5 degrees of freedom = 3 due to translation + 2 due to rotation.



$$\text{Total Energy} = \underbrace{\frac{1}{2}.m.v_x^2 + \frac{1}{2}.m.v_y^2 + \frac{1}{2}.m.v_z^2}_{\text{3 translational}} + \underbrace{\frac{1}{2}.I_y.w_y^2 + \frac{1}{2}.I_z.w_z^2}_{\text{2 rotational}}$$

b) **Law of equipartition of energy**:

Energy associated with each degree of freedom $= \frac{1}{2}.k.T$

Ex.1 Energy with Argon (Monoatomic) $= 3 \times \frac{1}{2}.k.T = \frac{3}{2}.k.T$

Ex.2 Energy with O_2 (diatomic) $= 5 \times \frac{1}{2}.k.T = \frac{5}{2}.k.T$

Q5. **Find**

a) C_v, C_p, γ **for monoatomic gas.**

b) C_v, C_p, γ **for diatomic gas.**

c) C_v, C_p, γ **for polyatomic gas.**

Ans.

<u>Monoatomic</u>

a) C_v

Internal energy, U $= N_A \times (K.E/atom)$

$= N_A \cdot \frac{3}{2} . \text{k.T}$

$$\boxed{U = \frac{3}{2} . R.T} \text{ ------ } (1)$$

As per definition, dU = (1). C_v.dT

$$\boxed{C_v = \frac{dU}{dT}} \text{ ------ } (2)$$

From (1) and (2)

$C_v = \frac{d}{dT}\left(\frac{3}{2} . R.T\right)$

$$\boxed{C_v = \frac{3}{2} . R}$$

C_p $\quad C_p = C_v + R$

$= \frac{3}{2} . R + R$

$$\boxed{C_p = \frac{5}{2} . R}$$

γ $\quad \gamma = \frac{C_p}{C_v} = \frac{5/2}{3/2} = 5/3 = 1.67$

<u>Diatomic</u>

b) C_v $\quad U = N_A \times (K.E/molecule)$

$= N_A (\frac{5}{2} . \text{k.T})$

$C_v = \frac{dU}{dT}$

$$\boxed{U = \frac{5}{2} . R.T}$$

C_p $\quad C_p = C_v + R$

$$\boxed{C_v = \frac{5}{2} . R}$$

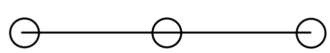
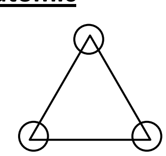
γ $\quad \gamma = C_p/C_v = \frac{7}{5} = 1.4$

$$\boxed{C_p = \frac{7}{2} . R}$$

c) **<u>Polyatomic Gas</u>**:

General formula for degrees of freedom of polyatomic gas.

$f = 3. A - R$ where A is number of atoms and R is number of relations.

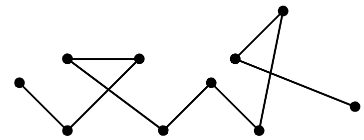
Case I **<u>Linear Triatomic</u>**	Case II **<u>Non-Linear Triatomic</u>**
	
$f = 3. A - R$	$f = 3. A - R$
$= 3 (3) - 2$	$= 3 (3) - 3$
$= 7$	$= 6$
$C_v = \frac{7}{2} . R$	$C_v = \frac{6}{2} . R = 3R$
$C_p = (\frac{7}{2} . R) + R = \frac{9}{2} . R$	$C_p = (C_v + R) = 4R$
$\Upsilon = C_p/C_p = \frac{9}{7}$	$\Upsilon = C_p/C_p = \frac{4}{3}$

Q6. **a) What is mean free path?**

b) Prove mean free path $\boldsymbol{\lambda = \frac{1}{\sqrt{2}.n\pi.d^2}}$?

Ans.a) **Mean Free Path**:

Path of a single gas molecule consists of a series of short zig zag paths of different lengths as shown in Fig. These paths of different lengths are called *free paths* of the molecules and their mean is called *mean free path*. We may define.



mean free path of gas molecules as the average distance travelled by a molecule between two successive collisions.

If $\lambda_1, \lambda_2, \lambda_3, \ldots\ldots \lambda_n$, are the successive path lengths travelled by a gas molecule in a total time *t*, then $\lambda_1 + \lambda_2 + \lambda_3 + \ldots\ldots \lambda_n = \bar{c}\, t$

Where $\bar{c}$ is mean speed of the molecules and *n* is number of collisions suffered by the molecule in *t* sec.

$$\therefore \quad \lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 + \ldots\ldots \lambda_n}{n}$$

b) **Mean Free Path**:

1. Volume swept per second = Area . $(\bar{c})$

$$= \frac{1}{4}\, \pi\, d^2\, (\bar{c})$$

2. No. of particles collected per second = (Volume swept/sec)(no. of particle/volume)

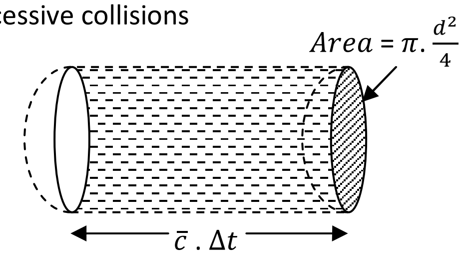
$$= \frac{1}{4}\, \pi\, d^2\, (\bar{c}) \times n$$

3. Average time between two collisions $= \frac{4}{\pi\, d^2\, (\bar{c})\, n}$

4. Mean free path = average distance between two successive collisions

= mean velocity x τ

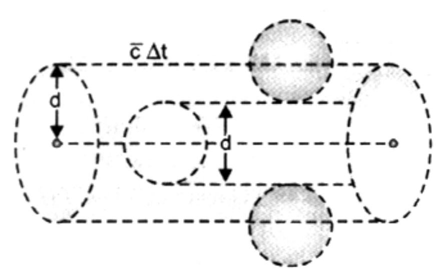
$$= \bar{c} \times \frac{4}{\pi\, d^2\, (\bar{c})\, n} = \frac{4}{n\, \pi\, d^2}$$



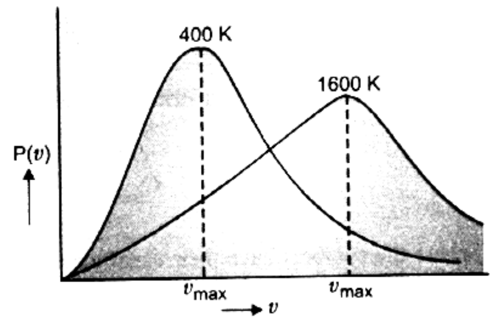
$$\boxed{\lambda \propto \frac{1}{n\, \pi\, d^2}}$$

$$\boxed{\boldsymbol{\lambda = \frac{1}{\sqrt{2}\, n\, \pi\, d^2}}}$$

Factor $\frac{1}{\sqrt{2}}$ is more accurate when we also account for motion of other particles having some speed.



c̄ Δt
d
d


400 K
1600 K
P(v)
v_max
v_max
v